The Weyl E

parity it was considered to be only of mathematical interest until 1956 when Lee and Yang proposed parity violation. The Weyl equation could then be used to describe neutrinos, which are massless[2]. It is now known that all spijn ½ particles, known as fermions, violate parity, thus they are all described by the Weyl equation before they acquire mass through the Higgs mechanism.

As we already know, $\boldsymbol{u}^{\dagger}\boldsymbol{u}$ is invariant under rotations, and $\boldsymbol{u}^{\dagger}\sigma\boldsymbol{u}$ transforms like a 3-vector. $\boldsymbol{u}^{\dagger}\boldsymbol{u}$ is not invariant under boosts. Instead we have,

$$\boldsymbol{u}^{\dagger}\boldsymbol{u} \rightarrow \boldsymbol{u}^{\dagger}\boldsymbol{e}^{\cdot}\boldsymbol{u} \simeq \boldsymbol{u}^{\dagger}(\boldsymbol{I} + \ \cdot)\boldsymbol{u} = \boldsymbol{u}^{\dagger}\boldsymbol{u} + \boldsymbol{u}^{\dagger}\boldsymbol{u} \quad (1)$$

where $\boldsymbol{\varphi}$ is infinitesimal. We can define the object $\omega^{\mu} \equiv (\boldsymbol{u}^{\dagger}\boldsymbol{u},\ \boldsymbol{u}^{\dagger}\sigma\boldsymbol{u})$ which transforms like a 4-vector.

We work with infinitesimal boosts in, without loss of generality, in the z direction. We have $\boldsymbol{u} \rightarrow (\boldsymbol{I} + \varphi\frac{\sigma_z}{2})\boldsymbol{u}$, which we can write as $\boldsymbol{u} = \frac{\sigma_z}{2}\boldsymbol{u}$, where the infinitesimal $\boldsymbol{\varphi}$ has been absorbed. We take the hermitian conjugate and get $= \frac{\sigma_z}{2}\boldsymbol{u}^{\dagger}$. Applying to $\boldsymbol{u}^{\dagger}\boldsymbol{u}$, we get,

$$(\boldsymbol{u}^{\dagger}\boldsymbol{u}) = (\boldsymbol{u}^{\dagger})\boldsymbol{u} + \tilde{\delta}\boldsymbol{u} \quad \sigma\ \boldsymbol{u}$$

We then take the derivative with respect to

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6 Majorana Mass term

We consider the possibility that there is another invariant term we can add to the Lagrangian for $\boldsymbol{u}$. Consider $\boldsymbol{u}^T\sigma_2\boldsymbol{u}$. checking , we transpose, $\boldsymbol{u} \rightarrow \boldsymbol{e}^{i\cdot}\boldsymbol{u}$ resulting in $\boldsymbol{u}^T \rightarrow \boldsymbol{u}^T\boldsymbol{e}^{i\cdot T}$. Note that ${}^T{}_2 = -{}_2$. Thus $\boldsymbol{u}^T{}_2\boldsymbol{u} \rightarrow \boldsymbol{u}^T\boldsymbol{e}^{i\cdot T}{}_2\boldsymbol{e}^{i\cdot}\boldsymbol{u} = \boldsymbol{u}^T{}_2\boldsymbol{u}$. It is invariant. It should be noted that $\boldsymbol{u}^T{}_2\boldsymbol{u}$ vanishes identically since σ_2 is an antisymmetric 2x2 matrix. However, to proof of which is beyond this paper, the components of $\boldsymbol{u}$ must be anticommuting Grassman numbers, $\boldsymbol{u}_1\boldsymbol{u}_2 = -\boldsymbol{u}_2\boldsymbol{u}_1$.

We then add the term $m\boldsymbol{u}^T{}_2\boldsymbol{u}$ to the Lagrangian for $\boldsymbol{m}$ constant. We also had the hermitian conjugate to keep the Lagrangian hermitian. We get,

$$= \boldsymbol{u}^\dagger(\qquad)\boldsymbol{u} \quad (m\boldsymbol{u} \quad \boldsymbol{u} \qquad \boldsymbol{u} \quad \boldsymbol{u})$$