In this paper, Feynman diagrams are presented as depictions of particle paths through spacetime. This is done in the context of the fourth order anharmonic modification of the free field theory. After presenting the rules that relate a Feynman diagram to its corresponding mathematical term, we provide a glimpse of functions in this context. To conclude the paper, we prove the logarithm property of the generating functional, which shows a deep relation between connected and disconnected diagrams.

In Quantum Field Theory, Feynman diagrams provide a visual representation of terms in the series expansion of probability amplitude quantities Equivalently, they illustrate how particles appear and, after propagating for some distance and possibly interacting with oth

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Feynman diagrams are most easily understood through a particular example. It is especially convenient to consider an anharmonic modification of the free field or Gaussian theory. In the pure free field model, the equation of motion is linear. This impllies that two independent fields coinciding in space, $\frac{1}{1}$ and $\frac{1}{2}$, will propagate without affecting each other. This is, each mode of vibration behaves as if the other were not present at all. Now, with the purpose of studying interaction between different solutions of our theory which is the mathematical requirement for our theory to indude collisions between particles-, we add the anharmonic potential term – $\frac{4}{4!}$ $^{-4}$ to the free field Lagrangian.

As usual, let (x) represent the source function, which indicates the locations in spacetime of sources and sinks of particles, (x) be the field, and be the characteristic

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This section is devoted to the step-by-

 $\begin{bmatrix} 2 & 2 \end{bmatrix}$.

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from $\frac{1}{1}$ to $\frac{1}{2}$, while $\frac{1}{1}$ ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) represents the scattering of two particles that start in $_{1},~_{2}$ and end up in $_{3},~_{4}.$ Therefore, by translational invariance, $^{(+)}(_{-1},\cdots,$ $)$ only depends on the differences between the arguments, − , in [1,), and not directly on the individual , because the probability of these kind of processes must obviously be a function of

We already mentioned that any diagram can be decomposed into its connected factors. This is:

$$
= \frac{1}{\sqrt{1-\frac{1}{2}}} \qquad () \quad [8]
$$

Where is a symmetry normalization factor which accounts for the number of different combinations of times s that can be combined to obtain . For identical factors, there exist ! identical rearrangements. So overall, counting over all, we find:

$$
= \qquad \qquad \left[9 \right]
$$

And the last three equations together render:

$$
Z(J) \t\t \t\t \frac{1}{I}() \t\t \frac{1}{I}() = \t\t [10]
$$

So is proportional to the exponential of all connected diagrams. But, by Eq. [6], is also proportional to the exponential of $($, $)$. It makes sense to impose the normalization constraint $Z(J = 0,) = 1$ which amounts, in physical terms, to disregard the diagrams with no sources, called vacuum diagrams. This sets the proportionality constant in Eq. [7], and hence in Eq. [10], to 1, if the sum is over all diagrams except to vacuum diagrams, while Eq. [6] reduces to the same statement for in place of the sum of \blacksquare . That does the job! We are led to the condusion that $\blacksquare(\blacksquare)$ constitutes indeed the set of connected diagrams, excluding vacuum ones. In other words, the ã