Inflationary Period: The Horizon Problem

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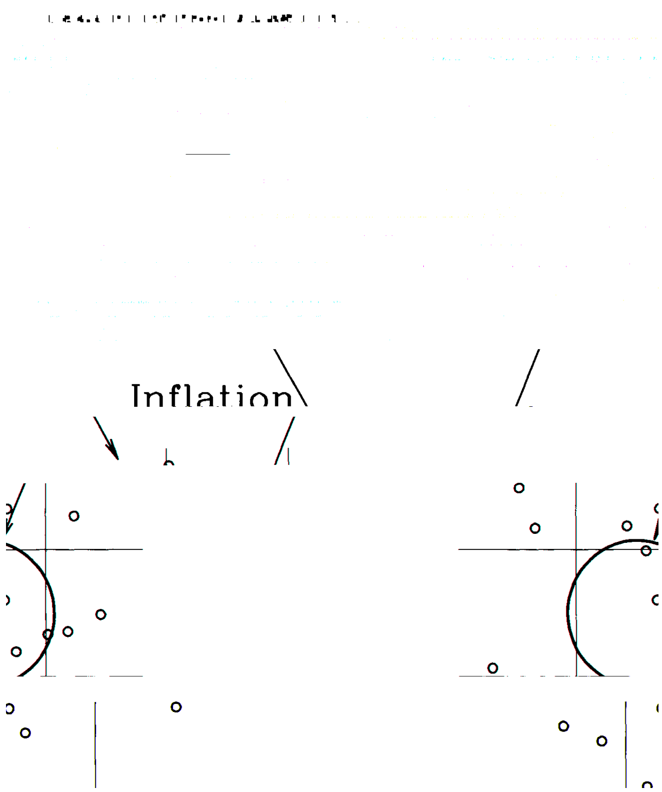
________ graphs a light cone (shaded region) in which we can view causal physics and measure what happens, whereas anything outside the light cone is unable to interact with us or other objects of the Universe on opposite sides of the light cone. The two squiggly-lines represent photons from the CMB, each beginning outside the realm of causality with us and one another. However, once they've traversed into the light cone, we are able to measure and view how they interact with other cosmological objects. It turns out that, despite being absurdly out of touch with one another, both photons will carry similar energies and thus obtain similar temperatures. This is the Horizon Problem: these photons could not possibly communicate to one another before they reached the light cone, or crossed within the Comoving Horizon, yet if they share similar temperatures Compton Scattering tells us they probably scattered from the same surface.

How is this possible? For years physicists tried to explain the phenomena to no avail, until some time in the 1980's the idea of "inflation" began working its way into the theory. Cosmologists today are not entirely sure the Inflation Model is truly the answer to the Horizon Problem, but another theory that solves many of the questions in the Horizon Problem has yet to be developed as much as Inflation has been, thus it is currently the "best guess."

The theory states that for a short period of time at the very genesis of the Universe, physical distances of microscopic scale blew-up to cosmic proportions. For instance, an Angstrom of space (about 10^{-10} meters) grew to Megaparsec scale (about $3*10^{6}$ lightyears) in the span of perhaps a few fractions of a second. This exponential growth is why the term "inflation" is used in descriptions of the theory, but just "how big" was this growth?

To attempt a derivation, the usage of Modern Cosmology by Scott Dodelson will be necessary. Most of the math is taken from Chapter 6, but other ideas such as the CMB and Boltzmann Equations come from earlier chapters. To start, remember that Inflation took place before anything else happened after the Big Bang. While the Universe was still a hot soup of photons, Inflation occurred and basically stretched the physical space where these photons existed. It expanded so quickly that the photons were unable to continue interacting with each other, or anything else for that matter, thus whatever they scattered off before Inflation decided their energies. ________ exemplifies this expansion.

________, Dodelson pg. 148



The photons didn't stop moving, however, thus once they traveled unabated through empty space to reach us now, we can measure them to have the exact energies they had back before Inflation occurred. These photons are the particle make-up of the CMB we see today, but we do not see them until they converge back into the Hubble Volume on , if we were to put ourselves at its center. To define the Hubble Volume, a few distinctions must be made. The Comoving Horizon has been mentioned, and a simple definition is that it represents the distance causal physics is allowed to occur across space for all of time. No matter what point in time in the Universe, matter dominated era or radiation dominated era or something else, the Comoving Horizon defines the total distance light can travel since the very genesis of the Universe at time $= 0$. (This definition comes from page 51 of Dodelson.)

Along with the Comoving Horizon, another cosmic distance of great importance is the Hubble Radius. This is what is meant by "Hubble Volume," in that the volume is a sphere of physical space with radius equal to the Hubble Radius.

The Hubble Rate is a ratio of how fast the Universe expands divided by the total distance it has expanded, giving it units of $(\quad)^{-1}$. Due to these units, inverting the Hubble Rate gives a measure of just how long the Universe has existed, with current estimates around 14 billion years. With these definitions, we can now define the , the expression of the same name below. This defines the distance particles can travel over one expansion time, or over the time it takes for the scale factor, $( )$, to double (Dodelson pg. 146).

$$= (( ) \cdot ( ))^{-1}$$

To connect this back to the theory of Inflation, remember that it is the idea that physical distances were much, much smaller than they are now, thus it makes sense to look at how the Comoving Hubble Radius needed to change during this period of Inflation to get the cosmological scale we see today. Referring back to , before inflation the Comoving Horizon resided within the Hubble Volume, but after Inflation it seems the Hubble Volume “shrunk,” thus allowing particles that were in causal contact before are no longer in contact afterwards. T

The primes in the equation are used to follow Integration rules, the variable integrated over cannot also be a limit to the integral, but what we see is that the Comoving Horizon is the logarithmic integral of the Hubble Radius. This implies then that shrinking the Hubble Radius corresponds to exponential growth. However, what would cause the Hubble Radius to shrink? During eras of radiation dominant or matter dominant epochs, the Hubble Radius will increase, so if it is to decrease, there must be some other dominant form of energy density. Some believe this to be “dark energy,” with representation in the Einstein Equations as the Cosmological Constant Λ, but the mechanism isn’t the focus of this paper.

Since we cannot base the evolution of the Hubble Radius on what type of energy dominates the Universe during Inflation, we can instead define it by how the scale factor, $( )$, evolves during this time period. The common way to do this is to look at how we defined the , $( )$, and hold the value itself to be constant. Rearranging then yields the following expressions:

$$\frac{\quad}{\quad} =$$

$$(( )) - (( )) = (-)$$

$$( ) = ^{(-)}$$

Here all subscript-e variables represent values at the end of Inflation, and the derivation is based on Dodelson page 147. Since we chose to hold constant, this restriction will also apply to the Comoving Hubble Rate and therefore also to the Comoving Horizon. This means any

It has been shown that expansion accelerates during Inflation, thus the term in parentheses must also be negative:

$$3 \quad + \rho < 0$$

$$< -\frac{\rho}{3}$$

This shows that for Inflation to occur, there must be a negative pressure in the Universe causing the accelerated expansion. This solidifies the idea that this era of the Universe's history is neither radi

Dodelson, S. (2002). ***Modern cosmology: anisotropies and inhomogeneities in the universe***. Academic.

NASA/WMAP Science Team. (2010, April 16). ***WMAP Inflation Theory***. NASA. https://wmap.gsfc.nasa.gov/universe/bb_cosmo_infl.html.