Free Field Theory and Propagators

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Abstract

We take the continuum limit of the mattress path integral model and show that it reduces to the classical field equation in the limit $\check{k} \to 0$. We then introduce the source function J(x) as a means to create and annihilate particles. We solve our path integral under the free field condition and get the Klein-Gordon Equation. We then define the propagator and evaluate it in free field theory using the method of contours.

This paper is adapted from Anthony Zee Quantum Field Theory in a Nutshell chapter I.3. The continuum limit is the basis for QFT. Once we have the path integral, we are ready to start making physical predictions. The path integral we get is only solvable for free field theory which can only be used to describe a single relativistic, massive particle. Studying the behavior and methodology of free field theory is useful for understanding for complex theories which describe scattering processes.

In our mattress model we derived the path integral for a single particle

$$Z = \int Dq(t)\exp[i \int_0 dt(½m\,(dq/dt)^2 - V(q))] \qquad (1)$$

We can easily generalize this to N particles with the new Hamiltonian

$$\sum \frac{1}{2m} \quad {}^2 + \quad (q_1, \quad {}_2, ..., \quad) \qquad (2)$$

We use a to label the particle's positions and momenta. Substituting back into the integral, we get

$$= \int \quad (t)\, ex[(i \int_0 \quad (\sum \tfrac{1}{2} m\, (\text{—})^2 - \quad (q_1, \quad {}_2, ..., \quad))] \qquad (3)$$

Which we simplify by defining the action

$$(q) = \int_0 \quad (\sum \tfrac{1}{2} m\, (\text{—})^2 - \quad (q_1, \quad {}_2, ..., \quad)) \qquad (4)$$

Thus we have

$$= \int \quad (t)e^{\ (q)} \qquad (5)$$

Note that the potential energy now includes interaction energy terms between particles which take the form $v(q_a - q_b)$ as well as the eternal potential energy terms which take the form $w(q_a)$. We take the special case where V has the form

$$(q_1, \quad _2, ..., \quad) = \sum \quad k \quad (q \qquad) \qquad ..$$

We now see why the variable q rather than x was used to denote position. In QFT x is a label rather than a dynamical variable, here the field   is the dynamical variable.

We can summarize the continuum limit in the table

(10)

Thus we get the continuum path integral for d dimensional spacetime

$$ = \int \quad \exp[i\int \quad (\tfrac{1}{2}(\partial)^2 - \quad ()] \qquad (11)$$

Note that we get quantum mechanics for d = 1.

We can take the classical limit of the path integral formalism as a sanity check. For convenience we have used units where ǩ = 1, we now return to SI units and put ǩ back into the path integral

$$ = \int \quad \exp (i/ǩ)$$

The above integral is only solvable when

$$() = \frac{1}{2}[(\partial)^2 - \quad {}^2{}^2] \quad (16)$$

We call this theory free or Gaussian theory.

We return to the propagator $(x -)$. Since it is the inverse of a differential operator, it is closely =