Faster-than-light Communication Cannot be Achieved Via Quantum Entanglement

Jiacan Yu
University of Rochester

December 15, 2021

Abstract

This paper explains why quantum entanglement does not provide a way of faster-than-light communication. To discuss the reason, the concepts of convex set and extremal point are introduced. Then it follows that density matrices form a convex set. This means that different ways of preparation could lead to the same mixed state. Hence, although via entanglement, one can immediately prepare a far away particle in different ways, that the prepared state is the same means that it is impossible to read the information.

1 Introduction

Quantum entanglement is one of the most fascinating and counterintuitive phenomena in quantum mechanics. If there is a pair of entangled particles, measuring the state of one of them, you would know the state of the other particle immediately, even if these two particles are on two opposite sides of the universe. This leads to some misconceptions in the public that we can send information faster than light using entangled particles. For example, in the science fiction *The Three Body Problem*, the aliens are able to establish real time communication with people on the Earth from 4 light years away via entangled particles. However, this is impossible. This paper will explain why quantum entanglement does not allow faster-than-light communication. The reason is the ambiguity in the preparation of mixed states. To illustrate this ambiguity, I will first define two useful mathematical concepts: convex set and extremal points.

2 Convex Set and Extremal Point

First, we define convex set. A subset S of a vector space is said to be convex if the set contains the straight line segment connecting any two points in the set. That is, for any $s_1 \in S$ and $s_2 \in S$, if $s(\lambda) = \lambda s_1 + (1 - \lambda)s_2$ are in S

Then we define extremal point. A point in a convex set S is an extremal point if it does not lie in any open line segment joining two distinct points in S. That is, the point s is called an extremal point if there do not exist two points s_1 and s_2 in S such that $s = \lambda s_1 + (1 - \lambda)s_2$ for any $0 < \lambda < 1$. Consider two sets in the x-y plane again, as shown in fig. 2, the extremal points of S are all the points on its edge. The extremal points of R are a, b, and c.

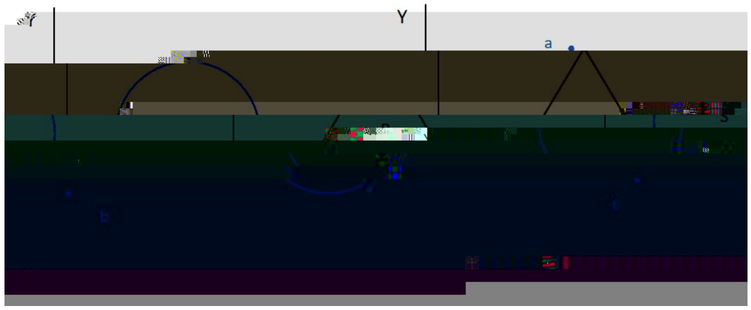

Figure 2: Examples of extremal points

Now let's go back to physics. In quantum mechanics, if a matrix ρ satisfies 1) ρ is self-adjoint, 2) ρ is

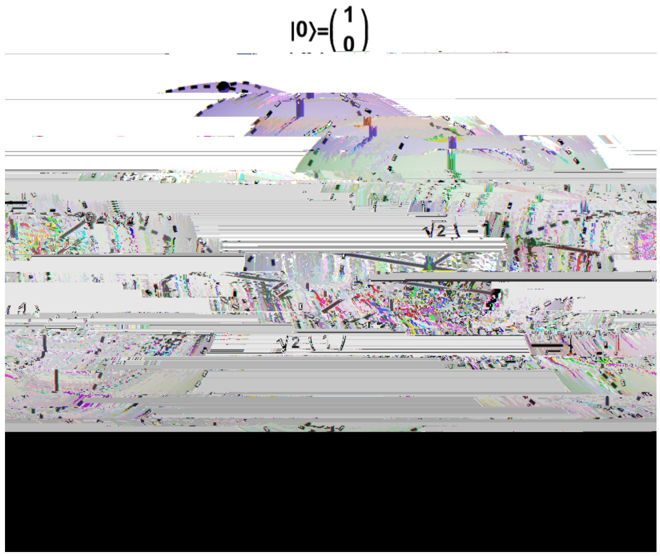

Figure 3:

and Alice stays on the Earth with the *A* particles. If Bob wants to send some information to Alice, he could measure his particles along either the x or the z axis one by one. This will correspondingly prepare