

# SIZE OF PROJECTION OF VECTOR SPACE OVER $Z_p^d$

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Abstract. The goal of the paper is to find the size of projection of vector space over  $Z_p^d$  by using the similar proof of Marstrand's projection theorem for one-dimensional projections.

## 1. Introduction

We discuss a special case of Marstrand's projection theorem in this paper. Let  $e$  be a unit vector in  $\mathbb{R}^n$  and  $E \subset \mathbb{R}^n$  a compact set. The projection  $P_e(E)$  is the set  $\{x \cdot e : x \in E\}$



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*Proof.* Suppose  $v(t) \in \mathbb{Z}_p$  and  $f(t) = f(x \cdot v) \in \mathbb{Z}_p^2$ .

Consider  $\frac{y}{t} = \frac{y}{t^2}$ . To make this equal to  $r$ , we must find  $t$  such that  $\frac{y}{r} = t^2$ .  
 Since  $y$  and  $r$  is not in  $Z_p$ ,  $\frac{y}{r} \notin Z_p$ .  
 Therefore  $V = S_1 \cup S_r$ .

Remark 4.1. We have the fact that  $\sum_{y \in Z_p^2} |\hat{E}(y)|^2 = p^{-2}|E|$

Theorem 4.5.  $|P_V(E)| = p \cdot \frac{1}{1 + \frac{p^2}{(p+1)|E|} - \frac{1}{p+1}}$ , if  $|E|$