Group Actions on Products of Spheres

i.e. an exact sequence in which P_k

Definition 3. *Let E and B be topological spaces, B connected, and $p : E \to B$ be a continuous surjection. Choose a point b B and let $F = p^{-1}(b)$. If for any point b B there is a neighborhood U_{b} B of b and a homeomorphism $\varphi : U_{b} \times F \to p^{-1}(U_{b})$, such that the following diagram commutes*

$$\begin{array}{ccc} U_{b} \times F & \xrightarrow{\varphi} & p^{-1}(U_{b}) \\ & \searrow^{\pi} & \downarrow p \\ & & U_{b} \end{array}$$

commutes, where $\pi : U_{b} \times F \to U_{b}$

Note that the added condition on the homeomorpshism $\varphi : U_b \times G \to p^{-1}(U_b)$ e□ectively asserts that φ

chains $C\ (EG)$. This action makes each $C\ (EG)$ into a G-module. We thus get a chain complex

$$\cdots \to C_2(EG) \to C_1(EG) \to C_0(EG) \to \mathbb{Z}$$

But since EG

G

$0 \to \hat{H}^{-1}($

6 Exponents

We will now use Tate cohomology to obtain results about the exponent of homology and cohomology groups, which we will then use to obtain the initial conjecture in certain special cases. Let M be a $\mathbb{Z}$-module with torsion. We let $\exp M$ denote the least positive $n \in \mathbb{Z}$ such that $nx = 0$ for all $x \in M$, and call $\exp M$ the *exponent* of M

For each k, let Z_k and ∂C_k denote the kernel and image of the boundary map $C_k \to C_{k-1}$, respectively. Then for each k, we get a short exact sequence

$$0 \to Z_k \to C_k \to \partial C_k \to 0$$

immediately from the definitions. Now, observe that $\hat{H}\ (G, C_k) = 0$ for all k, since each C_k is a free G-module. Thus, it follows from the above that $\hat{H}^i(G, \partial C_k) = \hat{H}^{i+1}(G, Z_k)$.

Using the definition of homology, we also get a short exact sequence

$$0 \to \partial C_{k+1} \to Z_k \to H_k(C) \to 0$$

for each k. Form the long exact sequence of Tate cohomology groups

$$\cdots \to \hat{H}^i(G, \partial C_{k+1}) \to \hat{H}^i(G, Z_k) \to \hat{H}^i(G, H_k(C$$

7 References

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2. Brown, Kenneth: Cohomology of groups

3. Duan, Zhipeng: Group actions on a product of spheres, emphasizing on simple rank two groups

4. Okutan, Osman; Yalcın, Ergun: Free actions on products of spheres at high dimensions