Algebraic properties of Artin-Mazur zeta function in positive characteristic

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Description

In the previous decades, a lot of effort was put into studying the properties of the Artin-Mazur zeta function (AMZF). For example, [Hin94] showed that rational maps of degree at least 2 acting on a Riemann sphere have rational AMZF and that its formula depends only on the number of parabolic cycles. More recently, [KH07] and [Ber21] showed that, in a measure-theoretic sense, almost every diffeomorphism has a divergent AMZF.

However, little is known about the AMZF in positive characteristic and the current progress is based on extremely specific cases. [Bri16] showed that the AMZF for Chebyshev polynomials, power maps, and Lattes maps is transcendental over $\mathbb{Q}(t)$, and separable additive map ρ has rational AMZF when $\rho^Q(0)$ is transcendental over $\mathbb{F}_p$. In his proof, Bridy relied on Christol's [Chr79] and Cobham's [Cob69] theorems, which relate algebraic field theory and automata theory. Inspired by Bridy's work, in this Honors Thesis we will investigate the algebraic properties of AMZF for map $f_p = x^2 + 1$ over $\mathbb{Q}(t)$ for fields of characteristic $p \geq 3$.

In our work, we will continue applying techniques from automata theory introduced by Bridy, however, we will also use some other concepts from automata theory described in [AS03]. In addition to this, we will introduce new ideas in order to get better understanding of number of periodic points of period n for the map $f = x^2 + 1$. Namely, we will apply the concept of the generalized Sen's

References

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