# A REMARK ON AN EQUATION OF WAVE MAPS TYPE WITH VARIABLE COEFFICIENTS

DAN-ANDREI GEBA

**Theorem 1.1.** *Let $3 \le n \le 5$ and assume $\partial^2 g \in L^2 L^\infty$. The Cauchy problem associated to* (9) *is locally well-posed in $H^s(\mathbb{R}^n) \times H^{s-1}(\mathbb{R}^n)$ for $s > \frac{n}{2}$.*

**Remark 1.2.** *The hypothesis on the regularity of the metric $g$ is related to the fact that, for both* (8) *and* (9) *, we have to control $X^{s,\theta}$ norms, which are in fact $L^2 L^2$ norms. A typical cross term to estimate is*

$$\|\partial^2 g \, u\|_{L^2 L^2} \lesssim \|\partial^2 g\|_{L^2 L^\infty} \|u\|_{L^\infty L^2}$$

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In the next section, for completeness, we will reintroduce the notations, de ni-

*then*

$$\|S \;\; r u\|_{L^p L^q} \; . \;\; 1 \;\; s \;\; +\tfrac{1}{2}(\tfrac{2}{p}+\tfrac{n-1}{q} \;\; \tfrac{n-1}{2}) d^{\frac{1}{2}} \;\; \tfrac{1}{2}(\tfrac{2}{p}+\tfrac{n-1}{q} \;\; \tfrac{n-1}{2}) \|u\|_{X^{s;}_{;d}} \tag{32}$$

Remark 2.8. *In the above embeddings , one can use also the index $q = 1$. We will rely in particular on the triplets*

$$( \;\; \frac{n}{2}; 1 ; 1 ) \qquad ( \;\; \frac{n \;\; 1}{2}; 2; 1 )$$

*noting that for $n = 3$ and $( \; ; p; q) = ( \;\; 1; 2; 1 )$, one loses in the previous bounds either a* ln *or* , *with* $> 0$ *arbitrary small. However, this loss is harmless because it is covered by the strict inequalities imposed on the exponents. Therefore*

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Then one can write

$$\Box_g u = \sum_{=1}^{1} \quad g_{>\sqrt{\ }} S \; u \;\; + \sum_{=1}^{1}$$

a $L^2L^2$ $L^1L^1$ one. We focus on (41). The rst two cross terms can be estimated directly, using (35):

$$^{s}\ 1 \quad k\,r\,S\ \ v\ \ S\ \ uk_{L^2} \ . \ \frac{\max f\ \ ;\ \ \frac{1}{2}g}{}\, k\,r\,S\ \ vk_{L^2L^\infty}kS\ \ uk_{X^{s;}_{;<}}$$

$$^{s}\ 1 \quad kS\ \ v\ \ r\,S\ \ uk_{L^2} \ . \ \max f\ \ ;\ \ \tfrac{1}{2}gkS\ \ vk_{L^2L^\infty}kS\ \ uk_{X^{s;}_{;<}}$$

More delicate terms appear when $g_{<\sqrt{}}$ acts on the product $S\ \ vS\ \ u$. The rst

(45) $$kS\ u\ S\ v\ k_{X^{s;}_{;}} \ . \quad ^{s+} \quad 2 \quad ^{\frac{n+3}{2}} \quad ^{2s}ku\ k_{X^{s;}}\ kv\ k_{X^{s;}}$$

(46) $$kS\ u\ S\ v\ k^2_{X^{s;}} \ . \quad ^{n} \quad ^{1+2} \quad ^{2s}kv\ k^2_{X^{s;}} \sum_{d=1} ku\ _{;d}k^2_{X^{s;}_{;d}}$$

It turns out that this is all that is needed to infer (42) (see also Proposition 3.7 in [2]).

Using the duality relation (29) and the fact that $s > \frac{n}{2}$, one can reduce (43) to

$$X^{s;} \quad X^{1 \quad s;1} \qquad X^{1 \quad s;1} \quad + L^2H^{2 \quad s}$$

which is then treated by considering decom0 6.9738 Tf 6T37h3todecom0-371v-371e. I71(then)h iscase Td 8isin