MATH 471 FINAL EXAM
DECEMBER 20, 2022

Instructions:

4. Let λ denote the restriction of Lebesgue measure on $\mathbb{R}$ to $(1, \infty) \subset \mathbb{R}$. For $f \in L^2((1, \infty), \lambda)$, let

$$G(y) := \int_1^\infty \frac{f(x)}{x+y}\, d\lambda(x); \quad y \geq 1.$$

Prove that G is well-defined, bounded and continuous on $(1, \infty)$.

5. Let V be a Banach space and V^* its continuous dual. Suppose $\{T_n\}_{n=1}^\infty \subset V^*$ is a sequence of bounded linear functionals on V such that, for every $f \in V$, $\lim_{n \to \infty} T_n f$ exists (the limit being taken with respect to the norm on F.)

Prove directly that there exists a bounded linear functional $T \in V^*$ such that $|T_n f - Tf| \to 0$ as $n \to \infty$ for all f.