1a) Prove that any finite field must have order a power of a prime p and for each n $\mathbb{Z}^+$, there is *one and only one* field of order p^n within a fixed algebraic closure of $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$.

b) Prove that all finite extensions of $\mathbb{F}_p$ are both normal and separable. Briefly explain why this implies that all finite extensions of all finite fields must be both normal and separable.

c) Prove that the Galois group over $\mathbb{F}_p$ of any finite extension of $\mathbb{F}_p$ is cyclic and give an explicit generator of such a Galois group, being sure to completely justify your answer. Briefly explain why this implies that all finite extensions of all finite fields must be cyclic.

d) If $E = \mathbb{F}_q = \mathbb{F}_{p^d}$ is a finite field with $q = p^d$ elements, and $K = \mathbb{F}_{q^r}$ is an extension of E of degree r, give a generator for $\mathrm{Gal}(K/E)$. You do *not* have to justify your answer to this part.

2) Show that if α is algebraic over a field k, then the multiplicity of α in its minimal polynomial $f(x) = \mathrm{irr}(\alpha, k, x)$ must be 1 if the characteristic is 0, and p^μ for some nonnegative integer μ if the characteristic is p [illegible]