1 *)* Define the n^{th} cyclotomic polynomial $\Phi_n(x)$ over an arbitrary field k where the characteristic of k is either 0 or a prime p not dividing n.

b) Prove that $\Phi_n(x)$ is irreducible over $\mathbb{Q}$.

) Give an example if possible, and briefly explain why your example works. If no such example exists, briefly explain why this is so.

) an integer n 5 satisfying the property that $\Phi_n(x)$ is b over $\mathbb{F}_p$ for all primes p not dividing n.

) an integer n 5 satisfying the property that $\Phi_n(x)$ is b over $\mathbb{F}_p$ for all primes p not dividing n.

) This relates to material in the book, not to a HW problem. To answer this question, you can use the additive form of Hilbert's Theorem 90 without proof.

Let k be a field in characteristic $p \neq 0$. Prove each of the following.

) Let K be a cyclic extension of k of degree p. Then $K = k(\alpha)$ for some α K that is a root of a polynomial x^p x a for some a k.

b) Conversely, for any a k, the polynomial x^p x a either has one root in k, in which case, all its roots are in k, or it is irreducible. Moreover, in the latter case, $k(\alpha)$ is Galois and cyclic of de