A II P /M 437 F
M 12, 2014

1) Let α and β be algebraic over F. Let $f(x) = \text{Irr}(\alpha, F, x$

Complex analysis 467 { Final exam

May 8, 2014

Please prove the following, justifying all statements.

1. Give two proofs of the fundamental theorem of algebra, with the first proof using Liouville's theorem, and the second using Rouche's theorem.

2. Let f be an entire function and suppose there exists constants $C > 0$ and D such that

$$|f(z)| \quad C|z|^n + D \quad \text{for all } z \in \mathbb{C}.$$

   Use Cauchy's estimates to prove that f is a polynomial of degree at most n.

3. Prove that

$$\int_0^{\infty} \frac{dx}{(x^2+1)^2} = \frac{}{4}$$

   making use of the contour defined by the line segment $[\ R; R]$ on the real axis and a large half-circle centered at the origin in the upper half-plane. (see picture)

4. Argue using Hadamard's factorization theorem that $e^z + z^2 + 1 = 0$ has infinitely many solutions in $\mathbb{C}$.

5. Let be a bounded simply connected domain in $\mathbb{C}$, and let and be two distinct points in . Let $_1$ and $_2$ be two conformal maps from $\ \to\ $. Suppose $_1(\ ) = \ _2(\ )$ and $_1(\ ) = \ _2(\ )$. Prove that $_1 = \ _2$. [Hint: one approach uses Schwarz's lemma.] (Side note: this is also true without the bounded condition.)

Math 453
Final Exam
May 3, 2014
9:00 - 12:00

Name : ______________________________

The exam consists of 5 questions.

Please read the questions carefully.

Show all your work in legibly written, well-organized mathematical sentences.

GOOD LUCK !!!

1. (20 pts) a) State the Regular Value Theorem.

b) Prove that the subset H of the Euclidean space $\mathbb{R}^3$ of all the points $(x; y; z)$ of $\mathbb{R}^3$ satisfying $x^3 + y^3 + z^3 \quad 2xyz = 1$ admits a C^1 2-manifold structure.

2. (20 pts) For the following vector elds X and Y and di erential forms and on $\mathbb{R}^3$, calculate the Lie bracket $[X; Y]$ and the Lie derivatives L_X and $L_{[X;Y]}(\wedge)$:

$$X = x\frac{\partial}{\partial x} \quad z^2\frac{\partial}{\partial y} \text{ and } Y = z\frac{\partial}{\partial y} + x^3\frac{\partial}{\partial z}, \quad = e^x dx + ydy + zdz, \quad = dx \wedge dy \wedge dz.$$

a) $[X; Y] =$

b) $L_X \quad =$

c) $L_{[X;Y]}(\wedge) =$

3. (20 pts) a) Determine whether the two-form $\omega = z\,dx \wedge dy$ is exact in $\mathbb{R}^3$.

b) Let M denote the embedded submanifold of $\mathbb{R}^3$ given by $M = \{z \quad x^2 \quad y^2 = 1\}$. Determine whether the restriction of ω to M is exact.

4. (20 pts) Let C be the punctured complex plane, C $f(0; 0)g$. Let $z = x + iy$ be

5. (20 pts) Show that the Laplacian $\Delta = d\delta + \delta d$ has the following properties :

a) Δ is self-adjoint, that is $\langle \Delta\omega, \eta \rangle = \langle \omega, \Delta\eta \rangle$.

b) A necessary and sufficient condition for $\Delta\omega = 0$ is that $d\omega$